Signature of the FFLO phase in the collective modes of a trapped ultracold Fermi gas

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We study theoretically the collective modes of a two-component Fermi gas with attractive interactions in a quasi-one-dimensional harmonic trap. We focus on an imbalanced gas in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase. Using a mean-field theory, we study the response of the ground state to time-dependent potentials. For potentials with short wavelengths, we find dramatic signatures in the large-scale response of the gas which are characteristic of the FFLO phase. This response provides an effective way to detect the FFLO state in experiments.

With the advent of ultracold atomic gases new states of matter are becoming experimentally accessible. Important developments have included the achievement of superfluidity in two-component Fermi gases [1], and the study of the effects of density imbalance in these systems[2]. These developments open up the prospect of the study of unconventional superfluid phases, in particular the inhomogeneous superfluid state of Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) [3]. This state has been under theoretical investigation since the 1960s, playing a central role in our understanding of superfluidity of fermions both within condensed matter physics and in particle physics [4]; yet to date no unambiguous demonstration of the FFLO state has been achieved.

While theory predicts the appearance of the FFLO phase in a bulk (3D) gas of ultracold fermions [5] the region of parameter space that it occupies is expected to be very small. It has been shown that this phase is greatly stabilised in (quasi-)1D geometries [6, 7, 8, 9]. Such geometries can readily be achieved in atomic gases by optical confinement[10], so experiments on ultracold Fermi gases will soon be in the regime where the FFLO phase can be studied. In view of this, it is important and timely to ask: what are the observable properties of the FFLO phase?

Existing proposals for the detection of the FFLO phase include the probing of noise correlations in an expansion image following release of the trap[9], and the use of RF spectroscopy to excite atoms out of the gas into other states [11]. In this Letter we show that characteristic signatures of the spatial inhomogeneity of the FFLO phase can be found in the collective mode response of the trapped gas. The method directly probes the intrinsic inhomogeneity of the FFLO phase – by the use of a perturbation by an potential with a short-wavelength spatial periodicity. An important feature of this method is that the signatures of the microscopic nature of the phase appear in the response on very *large* length scales (the scale of the atomic cloud). Consequently, the measurements of the response do not require high spatial resolution, and could be performed in situ without expansion of the gas.

We study a model of two-component fermions in 1D with attractive contact interactions, and unequal densities. For a homogeneous system, the exact groundstates are known from the Bethe ansatz [6, 7, 12]. Depending on the densities of the two components there are three phases that appear at $T = 0$: the unpolarised superfluid state; a fully polarised (therefore non-interacting) Fermi gas; and a partially polarised phase [6], associated with the FFLO state. This is a state that, within mean-field theory, has a local superconducting gap that oscillates in space. As has been discussed in detail by Liu et al.[13], mean-field theory provides an accurate description of the exact phase diagram for the 1D system, at least within the weak-coupling BCS regime. We have extended the mean-field theory to investigate the linear response and collective modes of both the trapped and untrapped imbalanced Fermi gas. As we shall discuss in detail later, this approximate theory correctly captures all qualitative features of the collective modes that are important for our purposes.

To effect numerical calculations, we study a discretized version of the problem (the attractive Hubbard model), for which the mean-field Bogoliubov-de Gennes (BdG) Hamiltonian is $\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i+1,\sigma}\hat{c}_{i,\sigma} + h.c.\right) + \sum_i\left(\Delta_i\hat{c}^{\dagger}_{i,\uparrow}\hat{c}^{\dagger}_{i,\downarrow} + h.c.\right) + \sum_{i,\sigma}W_{i,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma}$ where $\hat{c}^{(\dagger)}_{i,\sigma}$ are fermionic operators for species $\sigma = \uparrow, \downarrow$ on site i, $W_{i,\sigma} \equiv V^{ext}_i + U\langle\hat{c}^{\dagger}_{i,\bar{\sigma}}\hat{c}_{i,\bar{\sigma}}\rangle - \mu_\sigma$ (with V^{ext} the external potential and μ_σ the chemical potentials), and $\Delta_i \equiv U\langle\hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}\rangle$ is the local superfluid gap. J is the hopping parameter and U the on-site interaction strength ($U < 0$ is assumed throughout). We derive the linear response to external time-dependent perturbations, $\delta W_{i,\sigma}(t)$, by supplementing the (self-consistent) solutions of the above BdG equations with the random phase approximation (RPA) [14]. Divergences in the response appear at the frequencies of the collective modes of the system. The results we present are at sufficiently low particle density to be representative of the continuum limit. In the mapping to the continuum, we relate site i to position x via $x = ia$ and the mass is $m = \frac{\hbar^2}{2Ja^2}$. The interaction strength γ, defined as the ratio of the interaction energy density to the kinetic energy density, is given by $\gamma = -\frac{mg_{1d}}{\hbar^2\rho}$ [13] where $g_{1d} = \frac{U}{a}$, ρ is the total density of particles and $E_F \equiv \pi^2\rho^2 J/4$.

We discuss first the response of a homogeneous sys-

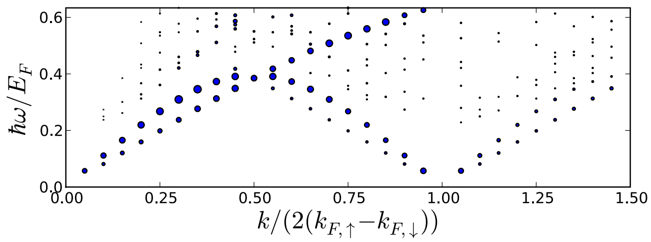

FIG. 1: Response [15] of a homogeneous system in the FFLO phase to a spin-asymmetric periodic potential (1). The area of the circle is proportional to the amplitude of the response. The polarisation $p \equiv \frac{k_{F\uparrow}-k_{F\downarrow}}{k_{F\uparrow}+k_{F\downarrow}} = 0.15$, and the interactions strength is $\gamma = 1.5$. Two gapless sound modes are seen to emerge around $k \simeq 0$ and $k \simeq k^* \equiv 2(k_{F_\uparrow} - k_{F\downarrow})$. The simulation was done on 270 lattice sites.

tem, with $V^{ext} = 0$ and periodic boundary conditions. We focus on the FFLO phase, with unequal average particle densities, for which the self-consistent mean-field solution has an oscillating gap $\Delta(x)$ with wavelength $\lambda_\Delta = 2\pi/(k_{F,\uparrow}-k_{F,\downarrow})$, where $k_{F,\sigma}$ are the Fermi wavevectors for the non-interacting imbalanced gas. We study the response to periodic perturbing potentials

$$\delta W_\sigma(x,t) = V_\sigma \sin kx \cos \omega t \tag{1}$$

We refer to the case $V_\uparrow = V_\downarrow = V_0$ as "spin-symmetric" and $V_\uparrow = -V_\downarrow = V_0$ as "spin-asymmetric" excitation.

The response of an imbalanced gas in the FFLO phase to a spin-asymmetric modulation is shown in Fig. 1. At low frequency and long wavelength there appear two distinct sound modes with different velocities [16, 17]. Within mean-field theory, the appearance of these two gapless modes arises from the fact that the FFLO phase breaks both gauge symmetry and translational symmetry. (We later discuss these properties beyond mean field theory.) An analysis of the two low-frequency modes shows that they are of mixed character, involving spatial oscillations of both density and spin-density. This is expected from the breaking of time reversal symmetry by the imbalance[17], and is consistent with the results (in Fig. 1) that both modes can be excited by a purely spin-asymmetric perturbation, $V_\uparrow = -V_\downarrow$.

In Fig. 1 it is clear that linear sound modes emerge also around the point $k = k^* \equiv 2(k_{F\uparrow} - k_{F\downarrow})$. Within mean-field theory, this too can be understood as a consequence of the broken translational symmetry of the FFLO phase. This leads to a Brillouin zone structure for the collective modes, characterised by a reciprocal lattice vector of size k^*. Note that this is twice the value that one would expect from the translational periodicity of the gap, λ_Δ. To understand this, observe that, while the gap Δ has period λ_Δ, the densities $\rho_\uparrow(x)$ and $\rho_\downarrow(x)$ have periods $\lambda_\Delta/2$, and the mean-field ground state has an exact symmetry under the transformation $x \to x + \lambda_\Delta/2$, $\Delta \to -\Delta$. Applying this symmetry to the RPA response calculation leads to a generalised Bloch theorem for the collective modes. The wavevector k is found to be conserved modulo $(2\pi)/(\lambda_\Delta/2) = k^*$, consistent with the response in Fig. 1. For a spin-symmetric perturbation a similar qualitative response is found. However, the response close to $k = k^*$ is smaller than for the spin-asymmetric perturbation. We account for this by the fact that the periodic density of excess majority particles manifests itself more strongly in the difference of the two densities than in the sum of the two densities. We now turn to discuss the manifestations of this response for a trapped gas.

We have studied the attractive Fermi gas in a harmonic trap, with $V^{\rm ext}(x) = \frac{1}{2}m\omega_0^2 x^2$ and with unequal particles numbers $N_\uparrow \neq N_\downarrow$. According to Orso [6] a 1D imbalanced trapped Fermi gas can only be in one of two configurations: (a) A partially polarised phase in the middle, with a fully paired phase towards the edge; (b) a partially polarised phase in the middle, with a fully polarised phase towards the edge. These configurations appear within the self-consistent BdG mean-field theory[13]. An example of the case (a) is given in Fig. 2, and of the case (b) in Fig. 3, where lengths are measured in units of $N^{1/2}a_{\rm ho}$ (with $N = N_\uparrow + N_\downarrow$ the total number of fermions, and $a_{\rm ho} \equiv \sqrt{\hbar/m\omega_0}$ the oscillator length of the trap). The configurations in Figs. 2 and 3 are in the weak coupling BCS regime, having values of γ for which BdG gives qualitatively the right density profiles [13]. We have studied the response of the trapped imbalanced Fermi gas in both these regimes.

Perturbations with wavelength much longer than the size of the system couple via potentials $V_\sigma(x)$ that are proportional to x. We find that the response to such potentials is well described by two sharp modes, involving the dipolar oscillations of the density and spin. One of these modes – the "Kohn mode" – involves the in-phase motion of both spin components, and has frequency $\omega = \omega_0$, independent of the state of the system. This exact result holds also within RPA[18], and is recovered to high accuracy in our calculations, showing that discretisation effects are minimal. The other mode – the "spin-dipole" mode – involves the relative motion of the two spin species. For attractive interactions the frequency lies slightly above ω_0 (at $\omega_{\rm sd} = 1.33\omega_0$ and $1.28\omega_0$ for Figs. 2 and 3 respectively). While the frequency of the spin-dipole mode does depend on the state of the system, it is does not show strong features of the presence of the FFLO phase. Similarly, the breathing modes are insensitive to the microscopic nature of the atomic gas[7].

The most interesting features arise when one excites the gas by time dependent potentials (1) with short wavelengths. We find characteristic signatures of the microscopic nature of the trapped gas in the response of spin-dipole mode – i.e. on length scales much larger than the wavelength of the applied perturbation.[27] In Figs 2(b) and 3(b) we show the amplitude of the response of the

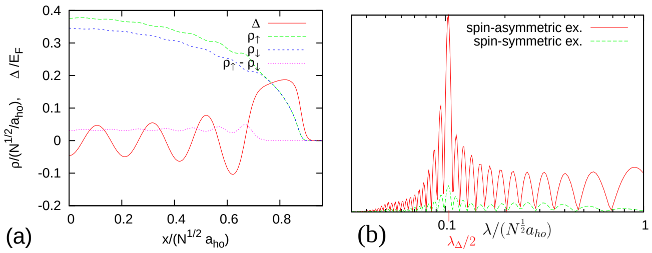

FIG. 2: (Colour online.) Configuration with fully paired phase towards the edge of the trap. (a): densities and the value of the superfluid gap Δ. (b): response of the spin-dipole mode to excitations of different wavelengths (arbitrary units). Here $p = 0.048$, $\gamma = 0.93$ (measured in centre), number of particles $N = 290$, lattice spacing $a = 3.3 \cdot 10^{-3} N^{\frac{1}{2}} a_{ho}$. The perturbing potential has a fixed amplitude, while the wavelength is varied.

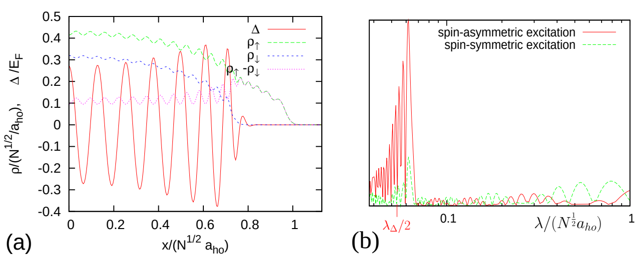

FIG. 3: (Colour online.) Configuration with fully polarised phase towards the edge. Here $p = 0.25$, $\gamma = 1.5$, $a/(N^{\frac{1}{2}} a_{ho}) = 4.7 \cdot 10^{-3}$, $N = 143$. Otherwise the same as fig. 2.

spin-dipole mode to spin-symmetric and spin-asymmetric periodic perturbations (1) as a function of wavelength $\lambda = 2\pi/k$.

For wavelengths λ that are short compared to the size of the cloud, $N^{1/2} a_{\mathrm{ho}}$, but large compared to λ_Δ, we find an oscillatory response as a function of λ in both Figs. 2(b) and 3(b). To understand this behaviour, we note that the spin-dipole mode can be excited when the two species experience a net difference in their acceleration. This difference is $a(k) = \frac{V_0 k}{m} \int [\rho_\uparrow(x)/N_\uparrow \mp \rho_\downarrow(x)/N_\downarrow] \cos(kx)\, \mathrm{d}x$ where the $\mp$ signs denote spin-symmetric/spin-asymmetric excitations. The difference in acceleration oscillates with k, in a manner that is largely controlled by the sizes of the clouds. For case (b) we assume, for simplicity, parabolic density distributions of size x_σ for the two spin components σ [28]. Defining $r \equiv \frac{x_\uparrow + x_\downarrow}{2}$ and $\delta \equiv \frac{x_\uparrow - x_\downarrow}{2}$, the condition for a maximum is $\sin kr \cos k\delta = 0$ for spin-asymmetric excitations, and $\cos kr \sin k\delta = 0$ for spin-symmetric excitations. For $\delta \ll r$ it follows that for spin-asymmetric (spin-symmetric) potentials maxima in the response come from the $\sin kr$ ($\cos kr$) term and the antinodes in the envelope come from the $\cos k\delta$ ($\sin k\delta$) term. Case (a) can be described by assuming a parabolic density distribution for the average density but where there is a constant difference between the two densities in the partially polarised region of size r_p. For spin-symmetric and spin-asymmetric excitations the condition for a peak is given by $\cos kr_p = 0$. The modulation of the peaks by this envelope structure allows us to distinguish the two imbalanced trap configurations (a) for which the two clouds have equal size $x_\uparrow = x_\downarrow$ and there is no modulation and (b) for which $x_\uparrow \neq x_\downarrow$ and a modulation appears, see figs. 2(b) and 3(b).

As the wavelength is further reduced, we find dramatic signatures of the presence of the FFLO phase, in both the cases (a) and (b) described above. Specifically, we find an unusually large response of the amplitude of the spin-dipole mode when the wavelength of the excitation becomes $\lambda = \frac{\lambda_\Delta}{2}$. This response can be understood in terms of coupling to the gapless modes at $k = \frac{4\pi}{\lambda_\Delta} = k^*$ of the homogeneous system (see Fig.1). A small deviation from k^* (of the order the inverse system size) allows mixing of this mode to the spin-dipole mode, causing the response at k^* to be apparent in the dipolar motion of the atomic cloud.

While the calculations we have presented are within RPA mean-field theory, as we now argue, the qualitative conclusions are valid more generally. Essentially, our results rely on the fact that in the unpolarised FFLO phase, there exists low-frequency (of order ω_0) response that is sharply peaked at a spatial wavevector k^*. In the above, this response was accounted for in terms of the broken translational symmetry of the mean-field state. As is well known, in a true 1D quantum system no continuous symmetries are broken. Thus, the broken (phase and translational) symmetries of the mean-field FFLO state can lead only to power-law decay of the respective correlation functions[19]. However, the qualitative features described above are the same as those expected from an exact treatment of the system. The transition from the (unpolarised) superconducting phase to the partially polarised phase is marked by the closing of the spin-gap, leading to a second gapless sound mode. This can be viewed as a Luttinger liquid representing the excess fermions[20]. These excess particles have density $\rho_\uparrow - \rho_\downarrow$, and thus a Fermi wavevector $(k_{F_\uparrow} - k_{F_\downarrow})$. Thus, there are two gapless collective modes, arising from the fully paired particles, and the liquid of excess majority spin particles. Furthermore, as in the general theory of Luttinger liquids[21] the spectral function of these excess majority spin particles will show gapless response at multiples of twice their Fermi wavevector. This is $2(k_{F_\uparrow} - k_{F_\downarrow}) = 4\pi/\lambda_\Delta$ which is precisely the wavevector k^* at which one finds the gapless response in mean field theory. Thus, the qualitative features of the collective excitation spectrum obtained in mean-field theory are fully consistent with those expected for the exact system.

Any distinction between the exact results and those of mean-field theory will be further reduced in the finite-size systems on which experiments can be performed, where

the distinction between long-range and power-law correlations becomes blurred. Indeed, one can expect that *density inhomogeneity* will play a more significant role than will the power-law decay of correlations. Since the densities of spin-up and spin-down particles are inhomogeneous, so too is (the local value of) λ_Δ, so one can expect smearing of the condition $\lambda = \lambda_\Delta/2$. However, even for the relatively small systems studied in Figs. 2 and 3, the collective mode response has a sharp onset at the value $\lambda = \lambda_\Delta/2$ at the centre of the trap. The appearance, in Fig. 3(b), of a large response at wavelengths slightly smaller than $\lambda_\Delta/2$ is associated with the non-constancy of λ_Δ, which decreases slightly towards the edge of the FFLO phase.

We propose that the signatures of Figs. 2 and 3 provide a convenient way to detect the FFLO phase in experiment. The sharp feature at $\lambda_\Delta/2$ appears only when the gap displays oscillatory behaviour, so is absent for a non-interacting gas and for temperatures when the FFLO phase disappears ($k_B T \simeq 0.1 E_F$ [22]).

In order to observe these signatures in experiments one needs to be able to create a variable wavelength optical lattice. This can be done using the technique used by Steinhauer et al. [23]. While the signal appears for a spin-symmetric perturbation, the response is larger for spin-asymmetric. Thus, any spin-dependence of the optical lattice will enhance the ability to distinguish the peaks associated with the oscillation of Δ from the background peaks. Optical lattices with spin-dependence have been created by Mandel et al. [24], and would be ideally suited to this purpose. There are two natural ways to find the response of the spin-dipole mode. One way is to follow precisely the approach of the calculation, and make the strength of the optical lattice time dependent, as in Ref.[25]. One can then selectively excite the spin-dipole mode by bringing the temporal oscillation of the lattice in resonance with the spin-dipole mode. Another way (similar to the approach used in Ref. [26]) is to apply a static periodic potential to the system, allow it to equilibrate, and then switch off this potential abruptly. This will excite collective modes of many frequencies from which the response of the spin-dipole mode can be obtained by Fourier transform.

In either case, we emphasise that the required measurements of particle density are on the length scale of the size of the cloud . Thus, our proposed method allows the probing of the microscopic physics by perturbing the system with short wavelengths while requiring only the measurement of densities on the length scale of the cloud.

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